AS and A Level Maths Formulae Sheet

Shapes

Area of Triangle	$\frac{1}{2}$ x base x height
Area of Parallelogram	base x height
Area of Rectangle	$length \times width$
Area of Trapezoid	$\frac{1}{2}$(sum of parallel sides) x height
Circumference & Area: Circle	$c = 2\pi r,\ A = \pi r^2$
Cuboid Surface area	$SA = 2xy + 2xz + 2yz$ Where $x, y,$ and z are side lengths
Cuboid Volume	$V = xyz$ where $x, y,$ and z are side lengths
Cylinder Surface Area	$SA = 2\pi rh + 2\pi r^2$ Note: Curved part: $2\pi rh$
Cylinder Volume	$V = \pi r^2 h$
Cone Surface Area	$SA = \pi rl + \pi r^2$ Note: Curved part: πrl where l is slant length
Cone Volume	$V = \frac{1}{3}\pi r^2 h$
Sphere Surface Area	$SA = 4\pi r^2$ Note: Hemisphere= $2\pi r^2 + \pi r^2 = 3\pi r^2$
Sphere Volume	$v = \frac{4}{3}\pi r^3$ Note: Hemisphere= $\frac{2}{3}\pi r^3$
Prism Volume	V =Area of cross section x height
Pyramid Volume	$V = \frac{1}{3} \times base\ area \times h$

Indices

Multiplication	$x^a \times x^b = x^{a+b}$ $(x^a)^b = x^{ab}$ $(cx^a y^b)^d = c^d x^{ad} y^{bd}$
Division	$x^a \div x^b = \frac{x^a}{x^b} = x^{a-b}$
Negative Powers	$x^{-n} = \frac{1}{x^n}$
Fractions	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ $\left(\frac{x}{y}\right)^{-n} = \frac{y^n}{x^n}$
Rational Powers	$a^{\frac{n}{m}} = \begin{cases} (a^{\frac{1}{m}})^n = (\sqrt[m]{a})^n \\ (a^n)^{\frac{1}{m}} = \sqrt[m]{a^n} \end{cases}$

Series

Arithmetic sequence: nth term	$u_n = a + (n-1)d$ where a =first term, d= common diff
Arithmetic sequence: sum of n terms	$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$ where a =first term, d= common diff, l=last term
Geometric sequence: nth term	$u_n = ar^{n-1}$ where a =first term, r= common ratio
Geometric sequence: sum of n terms	$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, r \neq 1$ where a =first term, r= common ratio
Geometric sequence: Sum to infinity	$S_\infty = \frac{a}{1-r},\ \|r\| < 1$ where a =first term, r= common ratio
Compound Interest	$FV = PV\left(1 + \frac{r}{100\ k}\right)^{kt}$ FV=future value PV=present value t=no. of years r=nominal annual interest rate k =no. of compounding periods per year
Binomial Theorem: integer powers	$(a+b)^n$ $=a^n + \binom{n}{1}a^{n-1}b + \cdots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$
Binomial Theorem: Fractional & Negative powers	$(a+b)^n$ $= a^n\left(1 + n\left(\frac{b}{a}\right) + \frac{n(n-1)}{2!}\left(\frac{b}{a}\right)^2 + \cdots\right)$
Binomial Coefficient	$\binom{n}{r} = nc_r = \frac{n!}{(n-r)!\,r!}$

Geometry

Straight Line: Equation (gradient means slope) Parallel⟹ same slope Perpendicular⟹ "flip fraction and change the sign" (slopes multiply to make -1)	• Slope intercept form: $y = mx + c$ • General form: $ax + by + d = 0$ • Point slope form: $y - y_1 = m(x - x_1)$
Straight Line: Gradient	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Distance between 2 points $(x_1, y_1), (x_2, y_2)$	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Coordinates of midpoint of $(x_1, y_1), (x_2, y_2)$	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Circles	$(x-a)^2 + (y-b)^2 = r^2$ centre (a, b), radius r

Quadratics

Quadratic Function: Solutions to $ax^2 + bx + c = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$
Quadratic Function: Axis of Symmetry	$f(x) = x^2 + bx + c \Rightarrow x = -\frac{b}{2a}$
Quadratic Function: Discriminant	$\Delta = b^2 - 4ac$ • > 0 (2 real distinct roots) • $= 0$ (2real repeated/double roots) • < 0 (no real roots)
Completing The Square $ax^2 \pm bx + c = 0$	$a\left(x \pm \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$
Max/Min Value	$c - \frac{b^2}{4a}$
Exponential and Logarithmic Functions	$a^x = e^{x \ln a}$, $\log_a a^x = x = a^{\log_a x}$ where, $a, x > 0, a \neq 1$
Exponentials & Logarithm Rules`	• $c \log_a b \Leftrightarrow \log_a b^c$ • $\log_a b = c \Leftrightarrow a^c = b, a, b, > 0, a \neq 1$ • $\log_a b + \log_a c \Leftrightarrow \log_a bc$ • $\log_a b - \log_a c \Leftrightarrow \log_a \frac{b}{c}$ • $\log_a b \Leftrightarrow \frac{\log_c b}{\log_c a}$ • Solving a power of x: log both sides if 2 terms or use substitution if 3 terms • Solving an exponential : ln both sides • Solving a logarithm: raise e both sides or write as $\log_e$ as procced as usual for log

Trigonometry

Sine Rule	Finding a side: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ Finding an angle: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
Cosine Rule	inding a side: $a^2 = b^2 + c^2 - 2bc\cos A$ Finding an angle: $A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$
Area of Triangle	$\frac{1}{2}absinC$
Degrees ⟷ radians	D to R: $\times \frac{\pi}{180}$ R to D: $\times \frac{180}{\pi}$
Length of an arc	$\frac{\theta}{360} \times 2\pi r$ (degrees) or $r\theta$ (radians)
Area of a Sector	$\frac{\theta}{360} \times \pi r^2$ (degrees) or $\frac{1}{2}r^2\theta$ (radians)
Small Angle Approximations	$\sin\theta \approx \theta$ $\cos\theta \approx 1 - \frac{\theta^2}{2}$ $\tan\theta \approx \theta$
Pythagorean identity 1	$\sin^2 x + \cos^2 x = 1$
Pythagorean identity 2	$1 + \tan^2 x = \sec^2 x$
Pythagorean identity 3	$1 + \cot^2 x = cosec^2 x$
Cofunction	$\cos x = \sin(90 - x)$ $\sin x = \cos(90 - x)$
Identity of $\tan x$	$\tan x = \frac{\sin x}{\cos x}$
Reciprocal	$\sec x = \frac{1}{\cos x}, \csc x = \frac{1}{\sin x}, \cot x = \frac{1}{\tan x}$
Double Angle	$\sin 2x = 2\sin x\cos x$ $\cos 2x = \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1 \Longrightarrow \cos^2 x = \frac{\cos 2x + 1}{2}$ $= 1 - 2\sin^2\theta \Longrightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$ $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$
Half Angle	$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$ $\cos\frac{x}{2} = \pm\sqrt{\frac{\cos x + 1}{2}}$ $\tan\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$
Compound Angle	$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$
Factor Formula: sum to product Note: For product t to sum re-arrange and let $\frac{A+B}{2}$ and $\frac{A-B}{2}$ equal your given angles and solve for A and B simultaneously	$\sin A + \sin B \equiv 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ $\sin A - \sin B \equiv 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\cos A + \cos B \equiv 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ $\cos A - \cos B \equiv -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

Vectors: 2D vectors $\begin{pmatrix} a \\ b \end{pmatrix}$ year 1 and 3D vectors $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ year 2

Vector Form	$ai + bj + ck \equiv \begin{pmatrix} a \\ b \\ c \end{pmatrix}$
Properties (addition/subtraction, multiplication and scalar product)	$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \pm \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} a \pm d \\ b \pm e \\ c \pm f \end{pmatrix}$ $\lambda\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \lambda a \\ \lambda b \\ \lambda c \end{pmatrix}$ $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad + be + cf$ (last formula not in syllabus but useful to know)
Magnitude of a vector	$\left\|\begin{pmatrix} a \\ b \\ c \end{pmatrix}\right\| = \sqrt{a^2 + b^2 + c^2}$
Unit Vector	Unit vector of $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{1}{\sqrt{a^2 + b^2 + c^2}}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$
Midpoint of $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\begin{pmatrix} d \\ e \\ f \end{pmatrix}$	$\left(\frac{a+d}{2}, \frac{b+e}{2}, \frac{c+f}{2}\right)$
Scalar Product (not in syllabus but useful to know)	$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \left\|\begin{pmatrix} a \\ b \\ c \end{pmatrix}\right\| \left\|\begin{pmatrix} d \\ e \\ f \end{pmatrix}\right\| \cos\theta$ where, θ is the angle between $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\begin{pmatrix} d \\ e \\ f \end{pmatrix}$
Angle Between 2 vectors This is just a re-arrangement of above. (not in syllabus but useful to know)	$\theta = \cos^{-1}\left(\frac{\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix}}{\left\|\begin{pmatrix} a \\ b \\ c \end{pmatrix}\right\| \left\|\begin{pmatrix} d \\ e \\ f \end{pmatrix}\right\|}\right)$
Vector Equation of a line (not in syllabus but useful to know)	$r = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \lambda\begin{pmatrix} d \\ e \\ f \end{pmatrix}$

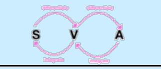
Probability and Statistics

Mean	If no frequency: $\bar{x} = \frac{\sum x}{n}$, If frequency: $\bar{x} = \frac{\sum fx}{\sum f}$
Variance	If no frequency: $\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2 = \frac{\sum(x-\mu)^2}{n}$ If frequency: $\sigma^2 = \frac{\sum fx^2}{\sum f} - \bar{x}^2 = \frac{\sum f(x-\mu)^2}{\sum f}$ Note: can also use the formula $\frac{S_{xx}}{n}$
Standard Deviation	$\sigma = \sqrt{variance}$
S_{xx}	$\sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$
Probability of event A	$P(A) = \frac{n(A)}{n(U)} = \frac{number\ of\ favourable\ outcomes}{number\ of\ possible\ outcomes}$
Complementary Events	P(A')=1−P(A) i.e. probabilities add to 1
Combined Events (Addition Rule)	P(A∪B)=P(A)+P(B)−P(A ∩ B)
Mutually Exclusive Events	P(A ∩ B) = 0 Addition rule becomes: P(A∪B)=P(A)+P(B)
Independent Events	P(A ∩ B) = P(A)P(B) Addition rule becomes:P(A∪B)=P(A)+P(B)−P(A)P(B) To find whether independent: Find P(A), P(B) and P(A ∩ B) and see whether the former 2 multiply to make the latter or show that P(A|B) = $P(A)$
Conditional "A given B"	$P(A\|B) = \frac{P(A \cap B)}{P(B)}$ If independent: $P(A\|B) = P(A)$
Bayes Theorem	$P(A\|B) = \frac{P(B\|A)P(A)}{P(B\|A)P(A) + P(B\|A')P(A')}$
Binomial Distribution Binompd (=) Binomcd (≤)	$x \sim B(n, p)$ E(X)=Mean= np, Var(X)= $np(1-p)$ $P(X = x) = \binom{n}{x}p^x(1-p)^x$
Normal Distribution Normcd (given x, want prob) Invnorm (given prob, want x)	$x \sim N(\mu, \sigma^2)$ Standardised variable $z = \frac{x - \mu}{\sigma}$
Interquartile Range	IQR= $Q_3 - Q_1$
Outliers	Any values $>$ UQ + 1.5(IQR) or $<$ LQ − 1.5(IQR)

Mechanics

SUVAT (5 formulae)	$v = u + at$ $s = \left(\frac{u+v}{2}\right)t$ $s = ut + \frac{1}{2}at^2$ $s = vt - \frac{1}{2}at^2$ $v^2 = u^2 + 2as$

Calculus (Differentiation and Integration)

Turning/Stationary Points (Max/Min)	Solve $\frac{dy}{dx} = 0$
Proving whether Max/Min	If $\frac{d^2y}{dx^2} > 0$ min and $\frac{d^2y}{dx^2} < 0$ max Or can do sign change test for $\frac{dy}{dx}$ using number line
Points of Inflection	solve $\frac{d^2y}{dx^2} = 0$
Increasing/Decreasing (use number line to solve)	To find where Increasing: solve $\frac{dy}{dx} > 0$ To find where decreasing: solve $\frac{dy}{dx} < 0$
Convex/Concave (use number line to solve)	To find where concave up/convex: solve $\frac{d^2y}{dx^2} > 0$ To find where concave down/concave: solve $\frac{d^2y}{dx^2} < 0$
Tangents and Normals	$y - y_1 = m(x - x_1)$ Differentiate to get m (tangent means ∥, Normal means ⊥)
Implicit	"every time we differentiate a y we write $\frac{dy}{dx}$"
Area between	curve & x axis: $\int_{x=a}^{x=b} y\,dx$ curve & y axis: $\int_{y=a}^{y=b} x\,dy$ (take + answer if neg) Between 2 curves: $\int_{x=a}^{x=b}$(top curve-bottom curve)dx Remember to split up if separate areas
Kinematics: 	Distance= $\int_{t_1}^{t_2} \|v(t)\|dt$, Displacement= $\int_{t_1}^{t_2} v(t)dt$ Velocity: $\int_{t_1}^{t_2} a(t)dt$ or $\frac{ds}{dt}$ Acceleration= $\frac{dv}{dt} = \frac{d^2s}{dt^2}$
Differentiation 1st Principles	$\frac{dy}{dx} = f'(x) = \lim_{h \to 0}\frac{f(x+h) - f(x)}{h}$
Chain Rule	$y = g(u), u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Product Rule	$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Derivatives	• $x^n \Longrightarrow nx^{n-1}$ • $(f(x))^n \Rightarrow n(f(x))^{n-1}f'(x)$ • $\ln(f(x)) \Rightarrow \frac{f'(x)}{f(x)}$ • $\sin f(x) \Rightarrow f'(x)\cos f(x)$ • $\cos f(x) \Rightarrow -f'(x)\sin f(x)$ • $e^{f(x)} \Rightarrow f'(x)e^{f(x)}$ • $a^{f(x)} \Rightarrow f'(x)a^{f(x)}\ln a$ • $\tan f(x) \Rightarrow f'(x)\sec^2 f(x)$ • $\sec f(x) \Rightarrow f'(x)\sec f(x)\tan f(x)$ • $cosec\, f(x) \Rightarrow -f'(x)cosec\, f(x)\cot f(x)$ • $\cot f(x) \Rightarrow -f'(x)csc^2 f(x)$ • $\sin^{-1} f(x) \Rightarrow \frac{f'(x)}{\sqrt{1-(f(x))^2}}$ • $\cos^{-1} f(x) \Rightarrow -\frac{f'(x)}{\sqrt{1-(f(x))^2}}$ • $\tan^{-1} f(x) \Rightarrow \frac{f'(x)}{1+(f(x))^2}$ • $\sec^{-1} f(x) \Rightarrow \frac{f'(x)}{f(x)\sqrt{(f(x))^2-1}}$ • $cosec^{-1} f(x) \Rightarrow -\frac{f'(x)}{f(x)\sqrt{(f(x))^2-1}}$ • $\cot^{-1} f(x) \Rightarrow -\frac{f'(x)}{1+(f(x))^2}$
Integrals	• $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$ • $\int \frac{1}{kx}dx = \frac{1}{k}\ln\|x\| + c$ • $\int \sin kx\,dx = -\frac{1}{k}\cos kx + c$ • $\int \cos kx\,dx = \frac{1}{k}\sin kx + c$ • $\int e^{kx}dx = \frac{1}{k}e^{kx} + c$ • $\int a^{kx}dx = \frac{1}{k\ln a}a^{kx} + c$ • $\int \sec^2 kx\,dx = \frac{1}{k}\tan kx + c$ • $\int \sec kx\tan kx\,dx = \frac{1}{k}\sec kx + c$ • $\int \operatorname{cosec} kx\cot kx\,dx = -\frac{1}{k}\operatorname{cosec} kx + c$ • $\int \operatorname{cosec}^2 kx\,dx = -\frac{1}{k}\cot kx + c$ • $\int \sec kx\,dx = \frac{1}{k}\ln\|\sec kx + \tan kx\| + c$ • $\int cosec\,kx\,dx = -\frac{1}{k}\ln\|cosec kx + \cot kx\|$+c • $\int \frac{1}{\sqrt{a^2-(bx)^2}}dx = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right) + c$ • $\int \frac{1}{-\sqrt{a^2-(bx)^2}}dx = \frac{1}{b}\cos^{-1}\left(\frac{bx}{a}\right) + c$ • $\int \frac{1}{a^2+(bx)^2}dx = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right) + c$
Integration by parts	$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$
Trapezium Rule h = $\frac{b-a}{number\ of\ strips}$	$\frac{h}{2}[y_0 + 2(y_1 + y_2 + y_3 + y_4 + \cdots) + y_n]$ Simply put, $\frac{1}{2}h[1st\ y + 2(middle\ y's) + last y]$
Newton Raphson	For solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Functions

Inverse	Replace $f(x)$ with y, swap x & y, solve for y
Composite	$fg(x)$ means plug $g(x)$ into $f(x)$
Odd and Even Functions	Even: $f(-x) = f(x)$ Odd: $f(-x) = -f(x)$
Transformations $af(bx + c) + d$ "anything in a bracket affects x and does the opposite"	a=vertical stretch sf a, b=horizontal stretch sf $\frac{1}{b}$ c=translation c units x direction, d=translation d units in y direction $f(-x)$=reflcn in y, axis $-f(x)$=reflcn in x axis

Linear: $y = mx + c$
Domain: x∈ ℝ
Range: y∈ ℝ

Quadratic: $y = \pm a(bx + c)^2 + d$
Domain: x∈ ℝ
Range: $y \geq d$ if min, $y \leq d$ if max

Exponential: $y = ae^{bx+c} + d$
Domain: x∈ ℝ (Hint: power of exp can be anything, so no restriction)
Range: $y > d$ if $a > 0, y < d$ if $a < 0$
(Hint: exp can't be zero)
Asymptote: $y = d$

Logarithm: $y = a\ln(bx + c) + d$
Domain: $x > -\frac{c}{b}$ (Hint: ln can't take a neg number so $bx + c > 0$)
Range: y ∈ ℝ
Asymptote: $x = -\frac{c}{b}$

Root: $y = a\sqrt{bx + c} + d$:
Domain: $x \geq -\frac{c}{b}$ (Hint: underneath root must be positive so $bx + c \geq 0$)
Range: $y \geq d$ if $a > 0$ and $y \leq d$ if a<0

Modulus $y = a\|bx + c\| + d$:
Domain: x ∈ ℝ
Range: $y \geq d : y \geq d$ if $a > 0$ and $y \leq d$ if $a < 0$

Note: Definition of $\|x\| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$

Rational: $y = \frac{a+b}{cx+d} + e$
Domain: x∈ ℝ, $x \neq -\frac{d}{c}$ (hint:denom≠0)
Range: y∈ ℝ, $y \neq \frac{a}{c} + e$
Asymptotes: $x = -\frac{d}{c}, y = \frac{a}{c} + e$
Note: often a and or e are zero

Trigonometry: $y = asin(bx + c)$+d
$y = acos(bx + c)$+d
Domain: $x \in \mathbb{R}$
Range: $-a + d \leq y \leq a + d$
Note: If asked to find values of a,b,c d
a = amplitude= $\frac{\max y - \min y}{2}$
$b = \frac{2\pi}{period}$ or $\frac{360}{period}$
d = principal axis= $\frac{\max y + \min y}{2}$
c =phase shift (plug in point to find)

Trigonometry: $y = atan(bx + c)$+d
Domain: $x \in \mathbb{R}, x \neq \frac{\pi}{2} + n\pi$
Range: $-\infty \leq y \leq \infty$

Inverse trig: $y = sin^{-1}x$
Domain: $-1 \leq x \leq 1$
Range: $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

Inverse trig: $y = cos^{-1}x$
Domain: $-1 \leq x \leq 1$
Range: $0 \leq x \leq \pi$

Inverse trig: $y = tan^{-1}x$
Domain: $-\infty \leq x \leq \infty$
Range: $-\frac{\pi}{2} < x < \frac{\pi}{2}$